NEWTONIAN HEATING ON MHD STAGNATION-POINT FLOW OVER A FLAT PLATE

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Abstract: A study on incompressible, steady magnetohydrodynamic (MHD) stagnation point flow of an electrically conducting fluid over a flat plate with variable thermal diffusivity and Newtonian heating has been considered. The governing partial differential equations were transformed using suitable similarity variables to couple nonlinear differential equations. The transformed equations are solved using the Runge-Kutta fourth order scheme with the shooting technique method. The effects of the various dimensionless flow parameters are presented in tables and graphs in terms of velocity and temperature profiles. Numerical computations for skin friction coefficient and Nusselt number are done. It was observed that thermal radiation parameter decreases the rate of heat transfer on the surface but increases in skin-friction coefficient while, increase in the viscosity and thermal diffusivity variation parameter increases both the skin-friction coefficient and rate of heat transfer. The results are in conformity with existing results.

Keywords: Body force, Brinkmann number, diffusivity, fluid flow, skin friction

Nomenclature

c_p	Specific heat capacity at constant pressure
g	Acceleration due to earth gravity
κ	Thermal conductivity $(W/(m.K))$
u	Velocity along x −axis (ms^{-1})
v	Velocity along y −axis (ms^{-1})
x	Coordinate along the plate
y	Coordinate normal to the plate
β	Thermal coefficient of volumetric expansion $(1/K)$
β_0	Magnetic field intensity
η	Similarity variable
ψ	Stream function
θ	Dimensionless temperature
ρ	Density of fluid (kg/m^3)
μ	Dynamic fluid viscosity $(N.s/m^2)$
U_∞	Stream velocity

Introduction

Magnetohydrodynamic (MHD) flow problems obviously owe relevance to various applications in industrial, manufacturing processes, engineering and science such as cooling systems for electronic devices, enhanced oil recovery, geothermal reservoirs, heat exchangers, cooling of nuclear reactors, etc. During the past years researchers have worked on several aspects of Newtonian heating and hydrodynamic boundary layer fluid flows. Makinde (2011) investigated second law analysis for variable viscosity hydrodynamic boundary layer flow with thermal radiation and Newtonian heating. Mahapatra and Gupta (2004) studied the boundary layer flow near the stagnation-point on a stretching sheet, where time dependence is also taken into account. Oyem *et al*. (2015a) considered free convective heat and mass transfer of reacting flow over a vertical plate and the effect of their thermophysical properties. Oyem *et al*. (2015b) considered combined effects of viscous dissipation and magnetic field on MHD over a vertical plate with thermal conductivity. Seth *et al*. (2015) investigated the unsteady hydrodynamic natural convection flow past an impulsively vertical plate with Newtonian heating in a rotating system. Sharma and Choudhary (2015) looked at the effect of radiation on MHD free convective flow of an electrically conducting fluid past a heated vertical porous plate embedded in a porous medium.

Several studies on MHD stagnation-point flow over a medium have been reported. Arthur and Seini (2014) studied MHD thermal stagnation point flow towards a stretching porous surface Sinha (2014) considered steady MHD stagnation point flow and heat transfer of an electrically conducting fluid over a shrinking sheet with induced magnetic field. Oyelami *et al*. (2015) looked into variable thermo-physical parameter effects on natural convective heat and mass transfer of a gray absorbing-emitting fluid flowing past an impulsively started vertical plate under the action of transversely applied magnetic field in the presence of chemical reaction. Oahimire and Olajuwon (2013) researched into hydromagnetic flow of a viscous fluid near a stagnation point on a stretching sheet with variable thermal conductivity and heat source/sink. Salem and Fathy (2012) investigated the effect of variable viscosity properties on MHD heat and mass transfer flow near a stagnation point towards a stretching sheet in a porous medium with thermal radiation. Aziz (2009) looked at a similarity solution of laminar thermal boundary layer over a flat plate with a convective surface boundary condition. Babu *et al*. (2018) analysed the impact of variable properties on heat and mass transfer over a vertical cone filled with nanofluid saturated porous medium with thermal radiation and chemical reaction.

Egunjobi and Makinde (2017) examined the combined effects of thermophysical properties on mixed convective flow of an electrically conducting Casson fluid in a vertical channel. The unsteady magnetohydrodynamic flow of nanofluid with variable fluid properties over an inclined stretching sheet in the presence of thermal radiation and chemical reaction was studied by Mjankwi *et al*. (2019). The steady magnetohydrodynamic stagnation point flow of an incompressible viscous electrically conducting fluid over a stretching sheet was investigated by Sinha and Misra (2014), and many other scholars (Olanrewaju *et al*., 2011; Al-Odat & Al-Ghamdi, 2012; Makinde & Olanrewaju, 2012; Makinde, 2012a; Makinde, 2012b; Das, 2012; Mostafa & Shimaa, 2012; Kameswaran *et al*., 2013; Christian &Seini, 2014; Ozalp, 2015; Krupalakshmi *et al*., 2016; Prasannakumara *et al*., 2016; Sivasankaran *et al*., 2017; Pandit & Sarma, 2017; Agbaje *et al*., 2018; Jyoti, 2017; Oyem, 2018; Raza, 2019; Anantha *et al*., 2020; Fenuga *et al*., 2020; Khan *et al*., 2020;

Oyem, 2020; Yasin *et al*., 2020; Afify & Elgazery 2020; Lund *et al*. 2020).

The aforementioned literature have dealt more on Newtonian heating for MHD stagnation-point fluid flows over a stretching sheet with deficiency on a flat plate; hence, this study. The aim of this research therefore, is to investigate the influence of steady MHD stagnation-point flow of an incompressible, electrically conducting heat transfer fluid over a flat plate. This is an extension of Makinde (2011) to include combined effects of body force, variable thermal conductivity, heat source and thermal radiation influence on the flow field.

Mathematical Analysis

A steady two-dimensional, incompressible hydrodynamic boundary layer flow over a flat plate with variable viscosity and thermal conductivity of an electrically conducting fluid in the presence of body force, heat source and magnetic field is considered. It is assumed that the induced magnetic field of the electrically conducting fluid and electric field due to polarization of charges are negligible. The fluid at the upper side of the plate is exposed to Newtonian heating; while the lower surface is heated by convection from a hot fluid as shown in Fig. 1.

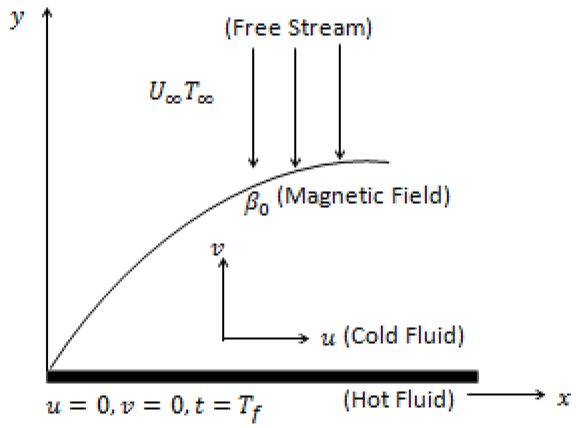

Fig. 1: Schematic of the problem

Base on the assumptions taken, Boussinesq's boundary layer approximation of the flow is then governed by the equations (Aziz, 2009; Makinde, 2011):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu(T)\frac{\partial u}{\partial y}\right)+g\beta(T-T_\infty) - \frac{\sigma\beta_0^2}{\rho}(u-U_\infty) \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(\kappa(T)\frac{\partial T}{\partial y}\right)-\frac{1}{\rho c_p}\left(\frac{\partial q_r}{\partial y}\right)+\frac{\mu(T)}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q(T-T_\infty)}{\rho c_p} + \frac{\sigma\beta_0^2}{\rho c_p}(u-U_\infty)^2, \qquad (3)$$

Where: u, v are the velocity components along the flow direction, U_∞ the free stream velocity, c_p is the specific heat at constant pressure, $\kappa(T)$ is variable thermal diffusivity, ρ the fluid density, σ is the fluid electrical conductivity, β is the coefficient of thermal expansivity, β_0 is the magnetic induction, g is the gravitational acceleration.

The boundary conditions for the velocity flow into the free stream are given as:

$$u(x,0)=0, \quad v(x,0)=0, \quad -\kappa\frac{\partial T}{\partial y}(x,0) = h_f\left(T_f-T(x,0)\right) \qquad (4a)$$

$$u(x,\infty)=U_\infty, \quad T(x,0)=T_\infty. \qquad (4b)$$

Where h_f is the heat transfer coefficient, T_f is the hot fluid at temperature, κ is the thermal conductivity coefficient. The fluid dynamical viscosity μ is assumed to be an inverse linear function of temperature (Lai & Kulacki, 1991) and thermal diffusivity κ is assumed to be a linear function of temperature given as (Charraudeau, 1975):

$$\mu(T)=\frac{\mu_\infty}{1+\gamma(T-T_\infty)} \qquad (5)$$

$$\kappa(T)=\kappa_\infty[1+\gamma(T-T_\infty)] \qquad (6)$$

Where: μ_∞ is the cold fluid viscosity, κ_∞ is thermal conductivity coefficient far from the plate surface and γ is a constant. By Rosseland's approximation (Sparrow, 1978), the radiative heat flux is given as:

$$q_r=-\frac{4\sigma^*}{3\kappa^*}\frac{\partial T^4}{\partial y} \qquad (7)$$

where, σ^* and κ^* are the Stephan-Boltzmann constant and mean absorption coefficient respectively. Assume the temperature differences within the flow are sufficiently small so that T^4 can be expressed as a linear function of temperature T using Taylor series about the free stream temperature T_∞ (Bataller, 2008), the result is the approximation

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \qquad (8)$$

Introducing stream function ψ, the continuity Eq. (1) automatically is satisfied using

$$u=\frac{\partial\psi}{\partial y} \quad and \quad v=-\frac{\partial\psi}{\partial x} \qquad (9)$$

Obtaining the similarity solutions toEqs. (1–4), we define an independent variable η and a dependent variable f in terms of the stream function ψ with dimensionless variables as:

$$\eta=y\sqrt{\frac{U_\infty}{\nu x}}, \quad \psi=\sqrt{\nu x U_\infty}\, f(\eta), \quad \nu=\frac{\mu_\infty}{\rho}, \quad \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty}. \qquad (10)$$

Substituting Eq. (10) into Eqs. (1–9), we obtained

$$\frac{d^3f}{d\eta^3}+\frac{(1+a\theta)}{2}f\frac{d^2f}{d\eta^2}-\frac{a}{(1+a\theta)}\frac{d\theta}{d\eta}\frac{d^2f}{d\eta^2}+Gr(1+a\theta)\theta - Ha(1+a\theta)\left(\frac{df}{d\eta}-1\right) \qquad (11)$$

$$\frac{d^2\theta}{d\eta^2}+\beta\left(\frac{d\theta}{d\eta}\right)^2+\frac{\beta Pr}{2}f\frac{d\theta}{d\eta}+\frac{\beta Br}{(1+a\theta)}\left(\frac{d^2f}{d\eta^2}\right)^2+\beta Pr Q\theta + \beta Br Ha\left(\frac{df}{d\eta}-1\right)^2 \qquad (12)$$

with dimensionless boundary conditions

$$f(0)=0, \quad \frac{df}{d\eta}(0)=0, \quad \frac{d\theta}{d\eta}(0)=Bi[\theta(0)-1], \quad \frac{df}{d\eta}(\infty) = 1, \quad \theta(\infty)=0. \qquad (13)$$

Where prime denotes differentiation with respect to η and $\beta=\frac{3Ra}{3Ra(1+a\theta)+4}$ is the thermal radiation influence. It is important to note that the local parameters Ha, Gr, Q and Bi in Eqs. (12–13) are functions of x defined as; $Ha=\frac{\sigma\beta_0^2 x}{\rho U_\infty}$ the local magnetic field parameter, $Bi=\frac{h_f}{\kappa_\infty}\sqrt{\frac{\nu x}{U_\infty}}$ is local convective heat exchange parameter, $Gr=\frac{g\beta x(T_f-T_\infty)}{U_\infty^2}$ is local Grashof number, $Pr=\frac{\nu\rho c_p}{\kappa_\infty}$ is Prandtl number, $Br=$

$\frac{\mu_\infty U_\infty^2}{\kappa_\infty(T_f-T_\infty)}$ is the Brinkmann number, $a=\gamma(T_f-T_\infty)$ is viscosity and thermal diffusivity variation parameter, $Ra=\frac{\kappa_\infty \kappa^*}{4\sigma^* T_\infty^3}$ is thermal radiation parameter and $Q=\frac{Q_0 x}{\rho c_p U_\infty}$ is the local heat generation parameter.

It is worthy of note to obtain the skin-friction coefficient and Nusselt number. Thus, the shear stress at the plate is given by:

$$\tau_w = C_f \frac{\rho U_\infty^2}{2} = \mu \frac{\partial u}{\partial y}|_{y=0}. \quad (14)$$

Where: μ is the coefficient of viscosity and the skin friction coefficient is defined as:

$$\frac{C_f}{2} = \frac{\tau_w}{\rho U_\infty^2}. \quad (15)$$

Using Eq. (10), we obtain

$$C_{f_x} Re_x^{\frac{1}{2}} = \frac{\partial^2 f}{\partial \eta^2}|_{y=0}. \quad (16)$$

Similarly, the heat transfer coefficient at the plate surface is given by;

$$q_w = -\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (17)$$

Where: κ is the thermal conductivity of the fluid. Thus, the Nusselt number is defined as:

$$Nu_x = \frac{x}{\kappa} \frac{q_w}{T_w - T_\infty}. \quad (18)$$

Using Eq. (10) and the heat transfer coefficient parameter, the dimensionless heat transfer rate is obtained as;

$$\frac{Nu_x}{Re_x^{\frac{1}{2}}} = -\frac{\partial \theta}{\partial \eta}|_{y=0}. \quad (19)$$

Numerical procedure

The coupled nonlinear ordinary differential Eq. (11) and Eq. (12) subject to the boundary conditions Eq. (13) are resolved numerically using Runge-Kutta fourth order scheme with shooting method. Eqs. (11–13) are reduced into a system of first order differential equations. Thus, we set $f=y_1$, $f'=y_2$, $f''=y_3$, $\theta=y_4$, and $\theta'=y_5$ such that,

$$\begin{aligned} y_1' &= y_2 \\ y_2' &= y_3 \\ y_3' &= -\frac{(1+a\theta)y_1 y_3}{2} - Gr(1+a\theta)\theta + \frac{a\theta' f''}{(1+a\theta)} + Ha(1+a\theta)(f'-1) \\ y_4' &= y_5 \\ y_5' &= -\beta(\theta')^2 - \frac{\beta Pr y_1 y_5}{2} - \frac{\beta Br (y_3)^2}{(1+a\theta)} - \beta Pr Q\theta - \beta Br Ha (f'-1)^2 \end{aligned} \quad (20)$$

subject to the initial conditions:

$$y_1(0)=0, y_2(0)=0, y_3(0)=s_1, y_4(0)=s_2, y_5(0)=bi\,[y_4(0)-1]. \quad (21)$$

Table 1: Comparison of results for $\theta(0)$ and $-\theta(0)$ for various values of Bi at $Ha=Br=a=Gr=Q=0$, $\beta=1.25$, $Pr=0.72$

Bi	$\theta(0)$			$-\theta(0)$		
	Aziz (2009)	Makinde (2011)	Present Study	Aziz (2009)	Makinde (2011)	Present Study
0.05	0.1447	0.1440	0.1439	0.0428	0.0428	0.0428
0.60	0.6699	0.6687	0.6687	0.1981	0.1988	0.1988
1.00	0.7718	0.7709	0.7785	0.2282	0.2291	0.2215

Table 2: Computation showing numerical results of $f''(0)$, $\theta(0)$ and $-\theta(0)$ for various values of Q, a, Gr and Ra at $Ha=1$, $Pr=0.72$, $Bi=Br=0.1$

	$f''(0)$	$\theta(0)$	$-\theta(0)$
$Q=0.0$	1.700951303026090	0.419868362340295	0.058013163766086
$Q=0.1$	1.834498645571800	0.493652463594279	0.050634753640603
$Q=0.2$	2.046782441578541	0.611623985500970	0.038837601449925
$Q=0.3$	2.442467536540596	0.834592252167878	0.016540774783215
$a=0.1$	1.865869913650765	0.484980424414534	0.051501957558533
$a=0.5$	1.989587275509568	0.455556768549795	0.054444323145023
$a=1.0$	2.120956921011591	0.430929778266363	0.056907022173358
$a=3.0$	2.530911757601666	0.381584686058069	0.061841531394195
$Gr=-0.1$	1.030123944209227	0.468761560739492	0.053123843926051
$Gr=1.0$	1.453297847254605	0.461460757932095	0.053853924206721
$Gr=1.5$	1.651127964108714	0.469722822361672	0.053027717763770
$Gr=2.0$	1.865869913650765	0.484980424414534	0.051501957558533
$Ra=0.7$	1.840816865045927	0.464693849794783	0.053530615020498
$Ra=3.0$	1.963756360266918	0.558939330552519	0.044106066944602
$Ra=5.0$	2.012714142937844	0.593243253536384	0.040675674645118
$Ra=10.0$	2.069576702992114	0.631659262264625	0.036834073773538

The numerical computation has step-size of $\Delta\eta = 0.001$ is chosen to satisfy the convergence criterion of10^{-14}. The plate surface temperature$\theta(0)$, skin-friction coefficient $f''(0)$ and Nusselt number $-\theta'(0)$ were computed and their numerical results are presented in Table 2 or different values of some governing parameters. While a comparison of existing research of Makinde (2011) and Aziz (2009) with the present study are presented in Table 1.

Results and Discussion

From Table 1, we observed that our numerical results the for plate surface temperature $\theta(0)$ and heat transfer coefficient in terms of local Nusselt number $\theta'(0)$ are in good agreement with that of Makinde (2011) and Aziz (2009) for different values of Bi. Similarly, the effects of some thermophysical parameters at constant values of $Ha = 1$, $Pr = 0.72$, $Bi = Br = 0.1$ on the skin-friction coefficient, plate surface temperature $\theta(0)$ and local Nusselt number variations are shown in Table 2. It was observed that increasing the heat generation parameter (Q), local Grashof number (Gr) and thermal radiation parameter (Ra), decreases the rate of heat transfer on the surface but increases the skin-friction coefficient. This is attributed to the physical fact that at higher radiation parameters, the fluid thermal boundary layer becomes thinner leading to increase in temperature gradient. Increase in the viscosity and thermal diffusivity variation parameter (a), increases both the skin friction coefficient and rate of heat transfer. This is due to the fact that, as convective heat transfers from the hot fluid on the lower side of the plate to the upper side, it increases due to Newtonian heating while, the fluid viscosity on the upper side of the plate decreases leading to an increase in velocity gradient and viscous dissipation (Makinde, 2011). Studying the effects of the thermophysical parameters involved on the flow, selected graphical results are presented in Figs. 2a – 2g and Figs. 3a – 3g.

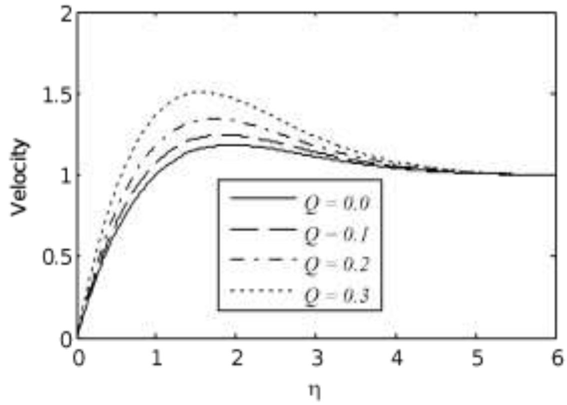

Fig. 2a: Velocity profiles for Q

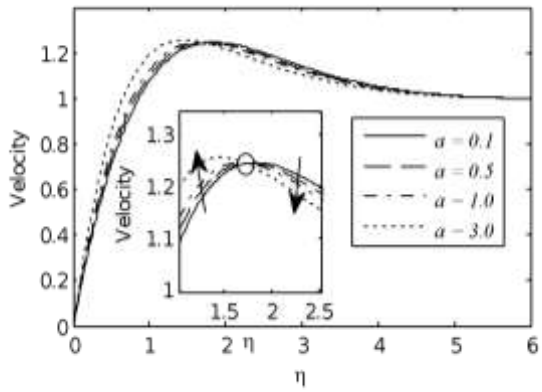

Fig. 2b: Velocity profiles for a

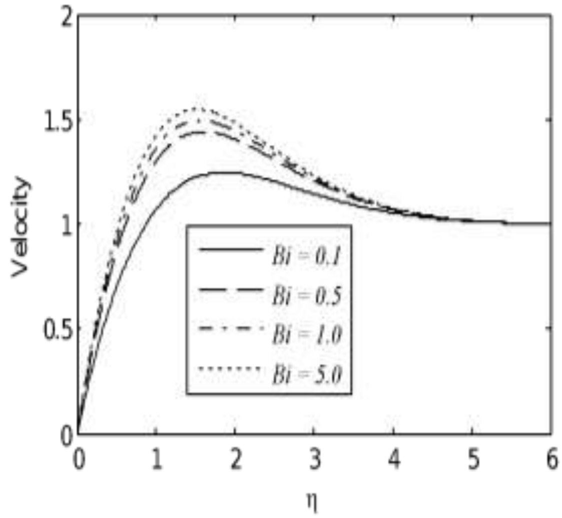

Fig. 2c: Velocity profiles for Bi

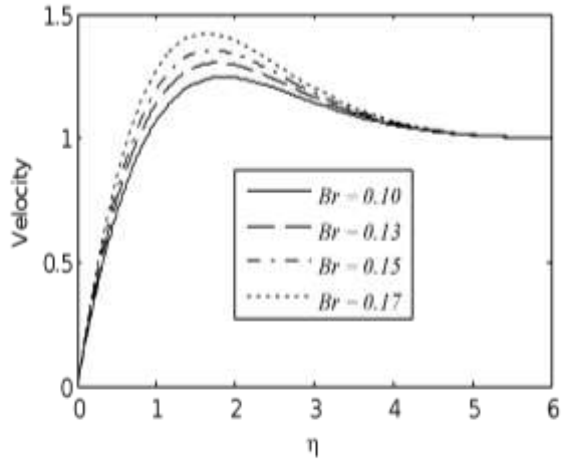

Fig. 2d: Velocity profiles for Br

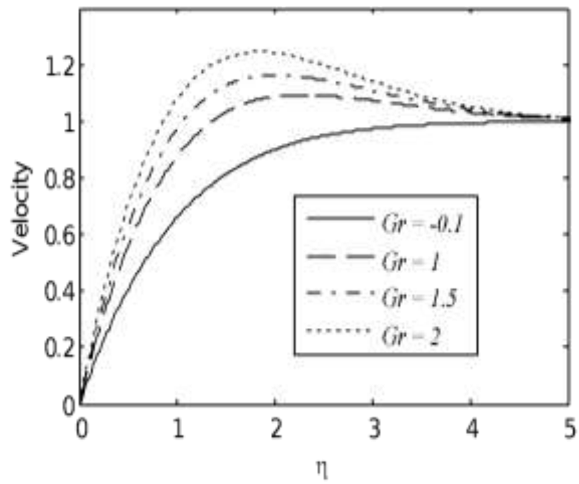

Fig. 2e: Velocity profiles for Gr

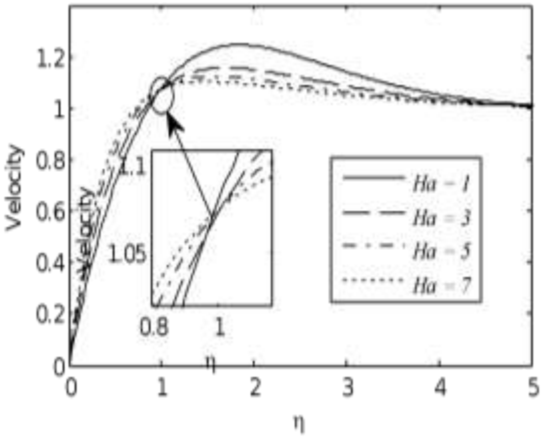

Fig. 2f: Velocity profiles for Ha

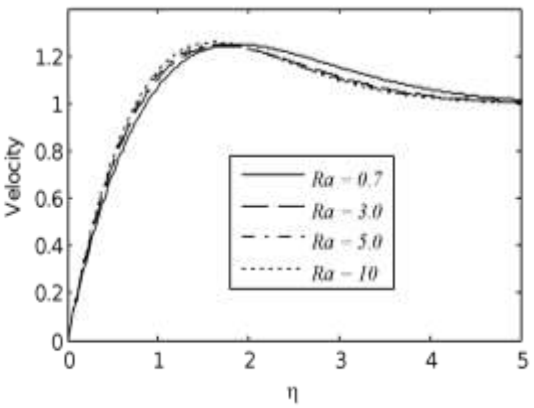

Fig. 2g: Velocity profiles for Ra

Effects of variation parameter on velocity profiles

Figures 2a to 2g respectively displays the effects of local heat generation parameter (Q), viscosity and thermal diffusivity variation parameter (a), local convective heat exchange parameter (Bi), Brinkmann number (Br), local Grashof number (Gr), local magnetic field parameter (Ha) and thermal radiation parameter (Ra) on velocity profiles. From Figs. 2b, 2f and 2g, the velocity profiles decreases along the plate and begins to increase to the free stream, satisfying the boundary conditions as the parameters; a, Ha and Ra increases. The effect of Fig. 2b on velocity profiles was due to the fact that the magnetic field creates a drag force that acts opposite to the fluid motion thereby, causing the velocity of the fluid to increase towards the plate surface. Also, from Fig. 2f, it was observed that as fluid viscosity decreases, the boundary layer becomes thinner and gradually increases in the fluid velocity gradient (Makinde, 2011). In Figs. 2a, 2c, 2d and 2e, it was observed that increasing values of Q, Bi, Br and Gr, tends to decrease gradually the velocity boundary layer towards the free stream.

Effects of variational parameter on temperature profiles

Effects of temperature profiles for the various values of a, Bi, Br, Gr, Ha, Ra and Q on the fluid flow are presented in Figs. 3a – 3g. It was observed from Figs. 3a and 3d, that increasing the values of local Grashof number (Gr) and variable viscosity and thermal diffusivity parameter (a), the temperature gradient of the flow, decreases gradually along the plate towards the free stream. Thermal boundary layer generates energy due to combined effects of viscous heating and Newtonian heating thereby, causing the temperature to increase as shown in Figs. 3b and 3c. It was also observed that temperature profiles increases with increasing values of Bi and Br. Similarly, increase in local magnetic field parameter (Ha), results in the increase in temperature profiles. This position gives rise to a resistive force known as Lorentz force of an electrically conducting fluid, making the fluid to experience a resistance by increasing the friction between its layers and thus, increase in temperature. In Fig. 3f, it was observed that as Q increases in value, temperature profiles also increase greatly away from the plate towards the free stream, satisfying the boundary conditions. A similar trend also plays on the effects of thermal radiation parameter (Ra). It was observed that thermal radiation parameter initially increases away from the plate but later gradually decreases in the thermal boundary layer thickness.

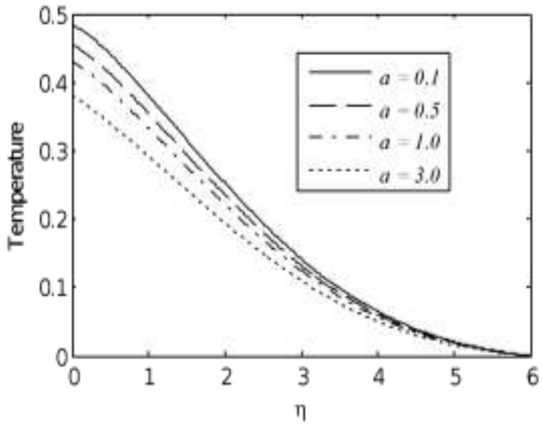

Fig. 3a: Temperature profiles for a

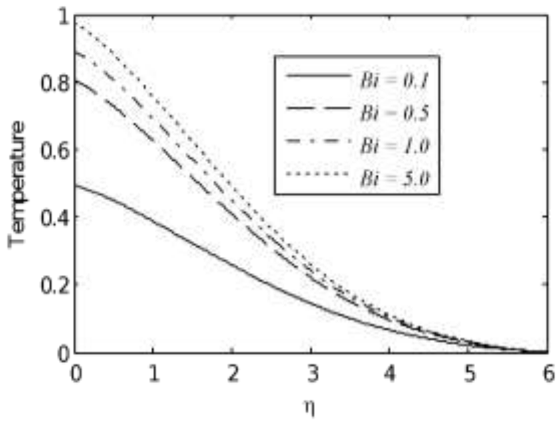

Fig. 3b: Temperature profiles for Bi

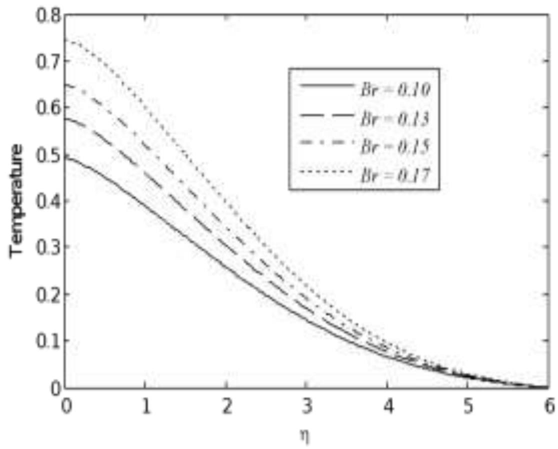

Fig. 3c: Temperature profiles for Br

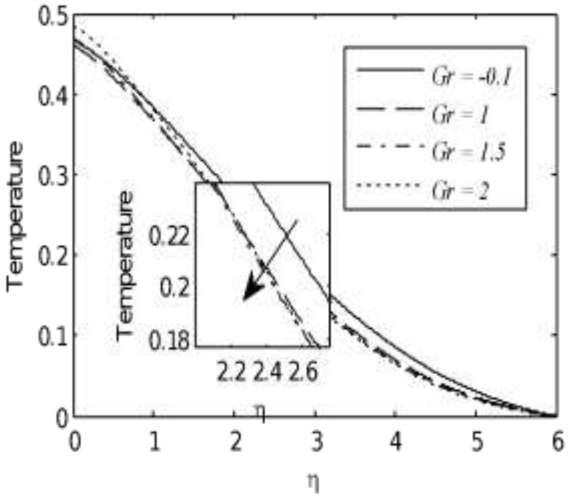

Fig. 3d: Temperature profiles for Gr

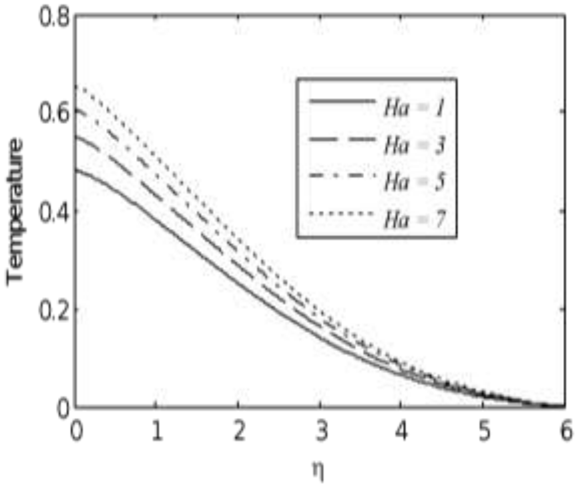

Fig. 3e: Temperature profiles for Ha

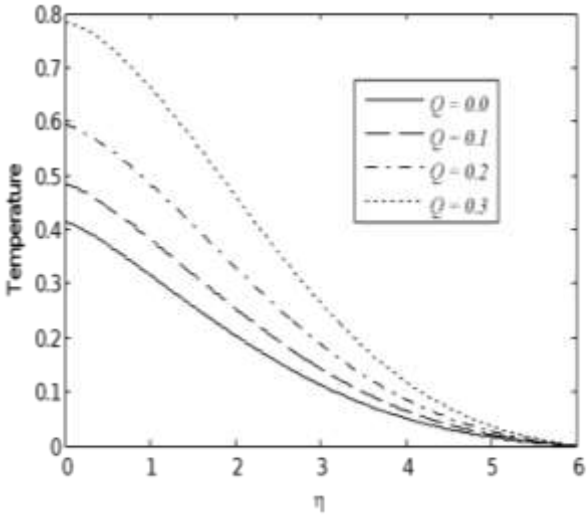

Fig. 3f: Temperature profiles for Q

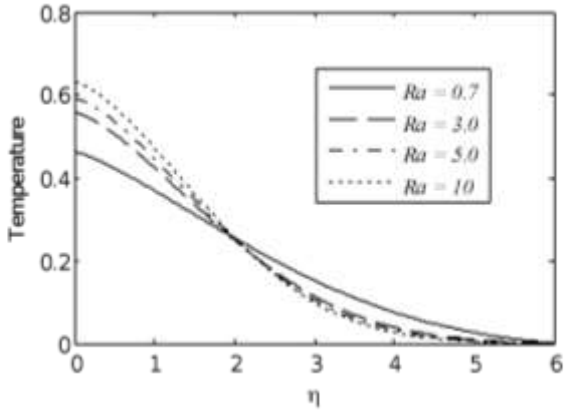

Fig. 3g: Temperature profiles for Ra

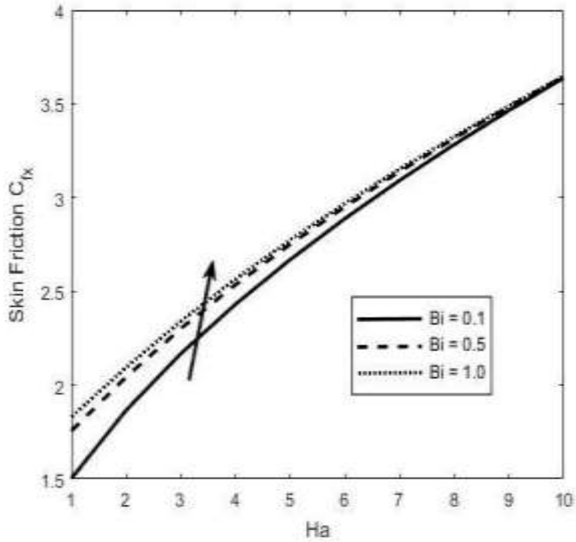

Fig. 4a: Variation of skin friction against Bi

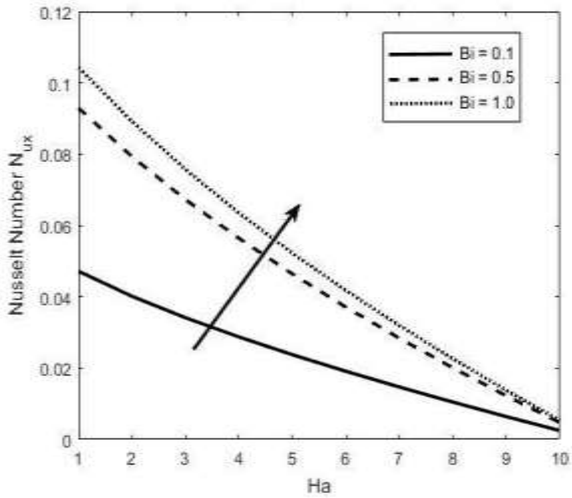

Fig. 4b: Variation of Nusselt number against Ha

The rate of skin friction coefficient and Nusselt number are presented in Figs. 4a and 4b. It is observed in Fig. 4a that skin friction coefficient increases from the wall and away towards a converging point from the boundary layer as Ha (local magnetic field parameter) increases in value. Similarly, in Fig. 4b, Nusselt number increases along the plate towards the thermal boundary layer with increasing values of Bi (convective heat exchange parameter).

Conclusion

A study of Newtonian heating and variable thermal diffusivity effects on MHD stagnation point flow over a flat plate was carried out. The velocity and temperature profiles for some governing parameters were obtained numerically by Runge-Kutta fourth order with shooting method. Their effects on velocity and temperature profiles were presented graphically. From the results obtained, the following conclusions are drawn:

1. The rate of heat transfer in terms of Nusselt number, decreases with increasing values of Q, Gr and Ra but decreases in skin friction coefficient and an increase in both skin friction and Nusselt number with increasing values of a.
2. The velocity boundary layer thickness decreases with increased values of Q, a, Bi, Gr and Br.
3. The thermal boundary layer thickness increases with Bi, Br, Ha, Q and Ra.

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Conflict of Interest

The authors declare that there is no conflict of interest related to this study.

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